

## Low-lying intrinsic structures in $^{254}\text{Es}$

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**Abstract.** Low-lying two-quasiparticle bandhead energies for the  $Z = 99$  odd-odd nucleus  $^{254}\text{Es}$  are evaluated using a simple phenomenological model with the inclusion of the residual  $p$ - $n$  interaction. Configurations of the intrinsic levels directly fed in  $^{254}\text{Es}$  from the parent  $^{258}\text{Md}$  are discussed in the light of this model. Our analysis predicts the occurrence of ten  $K \leq 5$  bandheads in  $^{254}\text{Es}$  with excitation energies  $E_x \leq 300$  keV. Structures of these as yet unidentified low-lying intrinsic levels and their expected locations are discussed in the light of available experimental information.

**PACS.** 21.10.-k Properties of nuclei; nuclear energy levels – 21.10.Hw Spin, parity, and isobaric spin – 21.60.-n Nuclear structure models and methods – 27.90.+b Properties of specific nuclei listed by mass ranges:  $220 \leq A$

### 1 Introduction

Nuclei in the transuranic region of the periodic table have been associated with several interesting properties. Available data [1] show that nuclei of this region in general have very low half-lives and low stability against spontaneous fission (SF) and  $\alpha$ -decay. However, nuclear species of the  $Z = 99$  element einsteinium (Es) are characterised by some unique features [2]. With the possible exception of  $^{257}\text{Es}$ , all the other known Es species show negligible SF branching resulting in fairly long-lived  $\beta$ -stable isotopes. Two einsteinium isotopes namely  $^{252}\text{Es}$  and  $^{254}\text{Es}$  are seen to have the longest half-lives for nuclei beyond the  $N = 152$  shell gap. Another interesting aspect of these nuclides is that the odd-odd species  $^{252}\text{Es}$  (471.7 d) and  $^{254}\text{Es}$  (275.7 d) are much longer lived than their odd- $A$  neighbours  $^{251}\text{Es}$  (33 h),  $^{253}\text{Es}$  (20.5 d) and  $^{255}\text{Es}$  (40 d) in sharp contrast to the general behaviour seen elsewhere. Despite such unique features, nuclear spectroscopic information on these nuclei is meager and is mostly based on several assumptions and plausibility considerations, particularly in the case of odd-odd nuclei. Even the ground-state spin parity is unambiguously known [1] for only 3 of the 20 Es species. Our recent reports [2, 3] attempted to characterise the lowest-energy states of nuclei in the  $Z = 99$  isotopic sequence with a view to look for as yet unidentified, long-lived isomeric species. Revised spin parity assignment for  $^{252}\text{Es}$  ground state was suggested [2] and preliminary results on the missing low-lying structures in  $^{254}\text{Es}$  were presented [3].

Experiments for synthesis of heavy elements, which lie beyond the reach of (HI,  $xn$ ) fusion-evaporation mode due to their almost vanishing fission barriers, are severely hampered in reaching the  $\beta$ -stable or neutron-rich domain in view of the inherent high neutron-proton ratio ( $N/Z \geq 1.55$ ). Any attempt to successfully produce such species using light projectile beams would require a relatively long-lived high- $Z$  target with a high  $N/Z$  ratio.  $^{254}\text{Es}$  ( $Z = 99$ ,  $N = 155$ ) with an observed half-life of 275.7 d is the heaviest  $\beta$ -stable, non-spontaneously fissioning nuclide that fits these criteria. Its use as a target in transfer reactions to produce still heavier Es isotopes and in experiments with  $^{18}\text{O}$  and  $^{22}\text{Ne}$  beams to produce  $n$ -rich Md isotopes with  $A = 256$  to 260 is well established. However, very little is presently known about its spectroscopic details.

Available experimental information for  $^{254}\text{Es}$  levels is obtained from  $\alpha$ -decay of the parent nucleus  $^{258}\text{Md}$  ( $T_{1/2} = 52$  d) [4–6]. Only two high angular-momentum bands—namely the  $K^\pi = 7^+$  ground-state band and 448 keV based  $K^\pi = 8^-$  band—have been directly identified so far. In addition, a 39 h  $2^+$  isomer state located around 84 keV in  $^{254}\text{Es}$  has been suggested [5, 6] as the  $\beta$ -decaying parent of  $^{254}\text{Fm}$ .

The  $I^\pi = 7^+$  assignment to the ground state of  $^{254}\text{Es}$  is consistent with low  $\beta^-$  and  $\epsilon$  branching to  $^{254}\text{Fm}$  and  $^{254}\text{Cf}$  nuclei [5, 6]. Based on the observed ground-state (g.s.) configurations of the odd-proton in  $^{253}\text{Es}$  and the odd-neutron in  $^{253}\text{Cf}$ , this state is designated [4–6] as the triplet member of the  $[p : 7/2[633] \otimes n : 7/2[613]]$  Gallagher-Moszkowski (GM) pair. A direct  $\alpha$ -branch from

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the parent  $^{258}\text{Md}$  (with an assumed 2qp configuration of  $[p : 7/2[514] \otimes n : 9/2[615]]$ ) was experimentally observed to proceed to the  $7^+$  g.s. of  $^{254}\text{Es}$ , though with a very low intensity and large hindrance factor (HF) [5,6]. The NDS evaluators [5,6], using the assigned structure argument, observe that no direct  $\alpha$ -feeding is expected to the g.s. from the parent  $^{258}\text{Md}$ , since it involves changes in both neutron and proton configurations. The weak observed  $\alpha$ -feeding of the  $7^+$  g.s. in  $^{254}\text{Es}$  is explained by them as indicative of probable configuration admixtures which we discuss later. The 80 keV  $I^\pi = 8^+$  level and the 170 keV  $I^\pi = 9^+$  level fit with their description as the next two rotational members of the  $7^+$  g.s. band with the rotational parameter  $A = 5.0$  keV. The low-lying isomeric state at 84 keV with  $I^\pi = 2^+$  [6] is the only low-spin state experimentally identified so far in  $^{254}\text{Es}$ . Since an electromagnetic decay from it to the  $7^+$  g.s. state is ruled out, the only depopulating mode would be through  $\beta^-$  or  $\alpha$ -emission. This state has been seen to possess a very high  $\beta^-/\alpha$  decay ratio [5] clearly indicating its preferred decay to the daughter nucleus  $^{254}\text{Fm}$ . The excitation energy of this low-lying isomeric state was evaluated from the  $Q(\alpha)$  value for its  $\alpha$ -decay to the 212 keV level in  $^{250}\text{Bk}$  [5].

Moody *et al.* [4] list the lowest HF (3.0) for the  $\alpha$ -transition feeding the 448 keV level in  $^{254}\text{Es}$  from the parent  $^{258}\text{Md}$ . Since the “likely” 2qp character of the parent g.s. [5,6] is  $8^- [p : 7/2[514] \otimes n : 9/2[615]]$  the same configuration is assigned to the “analogous”  $8^-$  state at 448 keV in  $^{254}\text{Es}$  which is fed through the lowest HF  $\alpha$ -branch. Experimental studies on reported gamma transition rates from the 448 keV level to the  $7^+$  g.s. band levels in  $^{254}\text{Es}$  with gamma energies 448 keV ( $448(8^-) \rightarrow \text{g.s.}(7^+)$ ), 368 keV ( $448(8^-) \rightarrow 80.1(8^+)$ ) and 277 keV ( $448(8^-) \rightarrow 171(9^+)$ ) are seen to differ considerably from the Bohr-Mottelson’s values for such transitions [4]. Moreover, NDS evaluators [5,6] have argued that if the suggested configurations are correct, then all these transitions should be forbidden since they would require both the proton and neutron configurations to change. Experimentally, the 368 keV gamma from the 448 keV  $8^-$  level is the most intense of all the gammas while the 448 keV transition is one of the most intense observed in  $^{254}\text{Es}$  [4–6]. This leads to the conclusion that large admixtures are expected in the 448 keV level and/or the g.s. itself.

Investigation of Ahmad *et al.* [7] in the region beyond  $N = 152$  shell gap predict several low-spin structures for these nuclei. Our present investigation is aimed at identifying and locating such missing structures in the  $^{254}\text{Es}$  level scheme.

## 2 Outline of formulation and procedure

The level structures of deformed odd-odd nuclei are normally described [8] using the rotor-plus-two-quasiparticle (2qp) formalism wherein each intrinsic state (bandhead) is assigned a 2qp Nilsson configuration  $(\Omega_p, \Omega_n)$  under the assumption that the residual neutron-proton interaction  $V_{np}$  can be treated as a perturbation. Thus the excitation

energy of a given 2qp configuration is written as

$$E(K : \Omega_p, \Omega_n) = E_p(\Omega_p) + E_n(\Omega_n) + E_{rot} + \langle V_{np} \rangle, \quad (1)$$

where  $E_p$  and  $E_n$  are usually taken from the observed [9, 10] excitation energies of the respective orbital in the neighbouring odd-mass ( $A-1$ ) isotope/isotone, and  $E_{rot}$  is the correction for the zero-point rotational energy. Each set of  $(\Omega_p, \Omega_n)$  orbitals gives rise to a doublet with the band quantum numbers  $K^\pm = |\Omega_p \pm \Omega_n|$ ; their relative energy ordering is decided by the empirical Gallagher-Moszkowski (GM) rule [11] which places the spin-triplet ( $\Sigma = 1$ ) state  $K_T$  lower in energy than its spin-singlet ( $\Sigma = 0$ )  $K_S$  counterpart. The separation energy of the  $K_T$  and  $K_S$  bandheads is denoted as the GM doublet splitting energy  $E_{GM}$ .

In view of the near-universal validity of the GM rule, Pyatov [12] chose an explicitly spin-spin interaction ( $\vec{\sigma}_p \cdot \vec{\sigma}_n$ ) term with a zero range, in addition to the spin-independent (Wigner) term, as  $V_{np}$  to describe  $E_{GM}$  and the odd-even spins splitting Newby [13] term  $E_N$  which appears only for the  $K = 0$  bands. Shortly thereafter, Struble *et al.* [14] chose to retain just these two data by writing

$$\langle V_{np} \rangle = - \left( \frac{1}{2} - \delta_{\Sigma,0} \right) E_{GM} + \delta_{K,0} (-)^I E_N. \quad (2)$$

Exhaustive surveys of the residual interaction contributions, using a variety of forces and form factors, were presented in 1976 by Boisson *et al.* [15] and by Elmore and Alford [16]. It may be noted that, in eq. (2), we adopt the sign convention of Elmore and Alford [16] for the Newby term. Based on their detailed comparison, Boisson *et al.* [15] concluded that the simple zero-range  $n-p$  force gives a fairly good description of  $E_{GM}$  values, but  $E_N$  values could not be fitted satisfactorily. Further they [15] pointed out that these studies yield no information about the Wigner term which plays a vital role in deciding the relative placement of different 2qp configurations. Also, the actinide region, though with scarce data then but steadily growing, was not adequately covered. Sood and Singh [17] focussed on these two aspects and derived analytical expressions for contributions from both the Wigner and the spin-dependent terms in a zero-range interaction. This formulation was extensively applied [17–19] to satisfactorily describe the known structures and to confidently predict the location of the as yet unidentified configurations in various odd-odd isotopes of  $^{93}\text{Np}$ ,  $^{95}\text{Am}$  and  $^{97}\text{Bk}$  species of the actinide region. Our recent investigation of  $^{101}\text{Md}$  levels [20] extended that domain into the trans-Berkelium region and the present report on  $^{254}\text{Es}$  levels is the first report on the  $A > 250$  odd-odd nuclei.

In contrast with these Nilsson-model-based phenomenological approaches, Bennour *et al.* [21] followed the *ab initio* theoretical approach to determine the ingredients of a Bohr-Mottelson unified model description from the Skryme III interaction through the adiabatic limit of a time-dependent Hartree-Fock-Bogolybov approximation. Their calculated level schemes for  $^{160}\text{Tb}$ ,  $^{174}\text{Lu}$  and  $^{238}\text{Np}$  were seen to be in good agreement. In addition,

**Table 1.** Expected low-lying 2qp bands in  $^{254}\text{Es}$  arising from single-particle orbitals with  $(E_p + E_n) \leq 200$  keV are shown in each cell. The top row and the first column list the odd-nucleon Nilsson configuration and its  $E_x$  from the neighbouring odd- $A$  nucleus for the valence  $n$  and  $p$ , respectively. Entries in each cell are  $K^\pi$  for  $K_T$  and  $K_S$  of each GM doublet in the first row and  $\Delta E(K_S - K_T)$  for each doublet (observed in  $^{250}\text{Bk}$ ) in the centre below it. Entries in the last column and bottom row pertain to data on the odd- $p$  and  $n$  orbitals used in the discussion on levels fed directly in  $^{254}\text{Es}$  from the  $\alpha$ -decay of  $^{258}\text{Md}$ , with the suggested experimental energy of the level of interest shown in italics in the line below along with the corresponding member of the GM pair.

N: $\Omega^\pi[Nn_3A] \rightarrow$ $\downarrow$ Z: $\Omega^\pi[Nn_3A]$	A: 7/2 <sup>+</sup> [613] 0 keV	B: 3/2 <sup>+</sup> [622] (70 keV)	C: 1/2 <sup>+</sup> [620] (100) keV	D: 9/2 <sup>+</sup> [615] 240 keV
X: 7/2 <sup>+</sup> [633] 0 keV	7 <sup>+</sup> 0 <sup>+</sup> 129.5	2 <sup>+</sup> 5 <sup>+</sup> 104.7	4 <sup>+</sup> 3 <sup>+</sup> 79.8	1 <sup>+</sup> 8 <sup>+</sup> <i>K<sub>S</sub> = 377</i>
Y: 3/2 <sup>-</sup> [521] 106 keV	5 <sup>-</sup> 2 <sup>-</sup> 49.0	0 <sup>-</sup> 3 <sup>-</sup> (80)	2 <sup>-</sup> 1 <sup>-</sup> 103.8	3 <sup>-</sup> 6 <sup>-</sup>
Z: 7/2 <sup>-</sup> [514] 370 keV	0 <sup>-</sup> 7 <sup>-</sup> <i>K<sub>S</sub> = 404</i>	5 <sup>-</sup> 2 <sup>-</sup>	3 <sup>-</sup> 4 <sup>-</sup>	8 <sup>-</sup> 1 <sup>-</sup> <i>K<sub>T</sub> = 448</i>

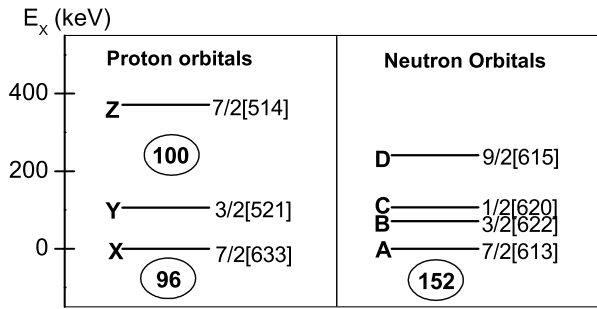
their calculated GM matrix elements were found to fit in nicely with the corresponding values deduced by Hoff *et al.* [22] in the phenomenological analysis for a wider set of data. Another theoretical approach [23–25] for the description of odd-odd deformed nuclei employs the extended quasiparticle-plus-phonon model (QPM). Here, the intrinsic Hamiltonian is taken as the sum of the axially symmetric average field  $H_{sp}$ , the short-range residual monopole pairing interaction  $H_{pair}$ , and a long-range residual interaction  $H_{QQ}$ . For the low-lying state it is considered sufficient to use  $H_{QQ}$  as a sum of separable isoscalar multipole-multipole  $Q_{\lambda\mu}^+ Q_{\lambda\mu}$  form. This formulation was successfully used [23] initially to deduce the microscopic quasiparticle structure of the states in  $^{166}\text{Ho}$  and  $^{168}\text{Ho}$  and then developed [24] for wider application. In the recent review article [25], QPM results are presented in comparison with those from the application of our eqs. (1)-(2) for all the nuclei from the rare-earth region. Mention may also be made of a critical, albeit comprehensive, compilation by Nosek *et al.* [26] of  $E_{GM}$  and  $E_N$  values for both the rare-earth and the actinide regions, and the use of the same to evaluate empirical parameters of a generalised residual  $n$ - $p$  interaction.

Based on a detailed comparison with presently known experimental level schemes [8] and with  $E_{GM}$  and  $E_N$  values [22] it may be concluded that modelling of the odd-odd deformed nuclei, particularly of the actinide region, may be satisfactorily pursued using eqs. (1)-(2). For instance, as reported by Sood *et al.* [8], the average deviation between the experiment and this model calculations is 32 keV for 13 bandhead energies in  $^{238}\text{Np}$  and just 17 keV for the 13 observed bands in  $^{250}\text{Bk}$ . We are presently extending its application to  $^{254}\text{Es}$ , which lies in the as yet unexplored domain of  $A > 250$  nuclei but is just an alpha-decaying parent of the above-mentioned well-studied  $^{250}\text{Bk}$  nucleus. Accordingly, it is reasonable to believe that this simple modelling is adequate to provide the guidelines for such explorations.

The single-particle levels of the  $Z = 99$  odd-mass neighbour of  $^{254}\text{Es}$ , namely  $^{253}\text{Es}$ , have been studied

through the  $\alpha$ -decay of  $^{257}\text{Md}$  as well as the  $\beta^-$ -decay of the ground-state of  $^{253}\text{Cf}$ . Data pertaining to these studies has been compiled in refs. [9,27]. The valence proton orbitals spanning the  $Z = 97$ –100 region are 3/2<sup>-</sup>[521] and 7/2<sup>+</sup>[633]. The proton configuration 7/2[633] is separated by less than 40 keV from the ground-state (g.s.) 3/2[521] configuration in the  $Z = 97$  nucleus  $^{247}\text{Bk}$ . In  $^{249}\text{Bk}$ , their ordering is reversed and the 3/2[521] orbital lies less than 10 keV above the g.s. 7/2[633] orbital. The situation reverses again in the  $N = 154$  nucleus  $^{251}\text{Bk}$  with the 3/2[521] configuration becoming the g.s. with the 7/2[633] orbital lying about 40 keV above the g.s. [9,10,27]. For the  $Z = 99$  Es isotopes, the 7/2[633] Nilsson orbital appears as the g.s. configuration in  $^{249}\text{Es}$  and  $^{253}\text{Es}$ . In  $^{251}\text{Es}$  3/2[521] appears as the g.s. with the 7/2[633] appearing barely 10 keV above it. It is hence quite clear that only these two orbitals play the dominant role in determining the low-lying structures for nuclei in the Bk-Es isotopic sequences. The 7/2[514] Nilsson orbital lies beyond the shell gap at  $Z = 100$ , well separated from the 7/2[633] orbital [7]. As such, it does not play an important role in our scheme of evaluation for levels with  $E_x \leq 300$  keV in  $^{254}\text{Es}$ . Similarly, the 5/2[642], 1/2[400] and 3/2[651] proton hole configurations lie below the shell gap at  $Z = 96$  with their separation from the 3/2[521] and 7/2[633] orbitals being at least 0.5 MeV. Hence, these orbitals too are not expected to play any significant roles in our evaluations. The relevant experimental proton single particle energies for the 7/2[633], 3/2[521] and 7/2[514] orbitals, taken from ref. [27] for the  $Z = 99$  odd-mass core nucleus  $^{253}\text{Es}$ , are shown in the first column in table 1, and are plotted on the left in fig. 1.

The neutron single-particle orbitals above the  $N = 152$  shell gap are 1/2[620], 3/2[622], 7/2[613], 11/2[725] and 9/2[615]. Among these, available experimental data [27] shows that the 1/2[620] orbital appears as ground state in the  $N = 153$  isotones ( $^{249}\text{Cm}$  and  $^{251}\text{Cf}$ ) and the 7/2[613] orbital is g.s. in the  $N = 155$  isotones ( $^{253}\text{Cf}$  and  $^{255}\text{Fm}$ ). From the remaining three neutron orbitals, data on the  $N = 153$  isotones clearly shows [9,10,27] the 3/2[622] or-



**Fig. 1.** Single-particle odd-proton and odd-neutron orbitals and their energies as observed in the nearest odd-mass neighbours. For protons, the nucleus is  $^{253}\text{Es}$  while for neutrons, they are  $^{251,253}\text{Cf}$  and  $^{255}\text{Fm}$ . The data is taken from refs. [7,27].

bit to be the closest to the two low-lying orbitals. The  $11/2[725]$  is observed in the  $N = 153$  nucleus  $^{251}\text{Cf}$  about 370 keV away from the g.s.  $1/2[620]$  while it is not observed at all in the  $N = 155$  nuclei [27]. For the  $N = 155$  nucleus  $^{253}\text{Cf}$ , the  $9/2[615]$  Nilsson orbital is seen separated by about 240 keV from the g.s.  $7/2[613]$  orbital. The nearest neutron orbital below the shell closure at  $N = 152$ , namely  $9/2[734]$ , lies about 0.5 MeV below the g.s. for the  $N = 153$  nuclei. Hence the three-neutron single-particle states of interest in our study are  $7/2[613]$ ,  $3/2[622]$  and  $1/2[620]$ . We take the  $7/2[613]$  single-particle level to be the g.s. configuration as observed in the nearest-neighbour  $N = 155$  nuclei  $^{253}\text{Cf}$  and  $^{255}\text{Fm}$ . No experimental data is currently available regarding the excitation energies of  $3/2[622]$  and  $1/2[620]$  orbitals in the  $N = 155$  nuclei. We therefore estimate the values (listed in the first row of table 1) from the observed relative locations of these orbitals [9,10,27] in the  $N = 153$  nucleus  $^{251}\text{Cf}$ . Accordingly the separation energies of the  $3/2[622]$  and  $1/2[620]$  orbitals from the g.s.  $7/2[613]$  is deduced to be about 70 keV and 100 keV, respectively. The separation energy for the  $9/2[615]$  orbital from the g.s.  $7/2[613]$  is 240 keV, as obtained from  $^{253}\text{Cf}$  ( $N = 155$ ) [27]. The situation in respect of the proton and neutron single-particle states of interest in the present study is summarised in fig. 1.

The input data used in our evaluation of the low-lying state in  $^{254}\text{Es}$  is listed in table 1. The listed experimental singlet-triplet energy splitting from neighbouring nuclei in each GM pair (listed in table 1) are taken from a previous compilation [8]. For the case of the GM gap between the  $0^-$  and  $3^-$  pair, where experimental data is not available, we consider the average of the singlet-triplet splitting energies between the pairs obtained from the two-quasiparticle configurations of the  $3/2[521]$  proton orbital with other relevant neutron orbitals [8]. For evaluation of the Newby shift  $E_N$  required for the  $p : 7/2[633] \otimes n : 7/2[613]$  combination, an empirical formula is employed [16] which is given below:

$$E_N = \frac{1}{2} [E_0(J) - E_0(J+1)] + \frac{\hbar^2}{2I} [(-1)^J(J+1) + a_p a_n \delta_{\Omega,1/2}]. \quad (3)$$

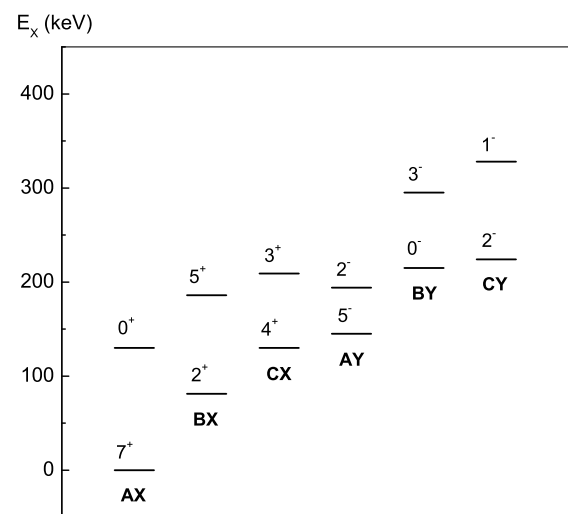
The same combination of  $p : 7/2[633] \otimes n : 7/2[613]$  appears in the  $N = 153$  nucleus  $^{250}\text{Bk}$  wherein the  $K = 0^+$  band arising out of the combination is experimentally observed [5] at 215.94 keV while the nearest rotational member of this band ( $1^+$ ) lies below it at 175.13 keV. We apply eq. (3) to these two lowest members of the  $K^\pi = 0^+$  band and evaluate the Newby shift to be 25 keV for this structure.

### 3 Results and discussion

Based on the discussion in the previous section, we list in table 1 the neutron and proton single-particle configurations and also the expected  $K^\pi$  for each member of the relevant 2qp GM pair. Figure 2 shows the predicted bandhead energy for each of the twelve bands expected as two-quasiparticle configurations evaluated using our eqs. (1)-(2) with the input data as given in table 1. The results are discussed individually for each band in the following subsections.

#### 3.1 Well-established levels

The  $7^+$  g.s. is identified with the triplet member from the  $p : 7/2[633] \otimes n : 7/2[613]$  configuration. No violation in the GM rule [11] is expected in this case; we predict the  $0^+$  singlet state of the above configuration to occur at about 130 keV above the g.s. This state remains to be experimentally observed. The  $2^+$  isomeric state at 84 keV is the only experimentally observed low-lying low-spin state in  $^{254}\text{Es}$ .  $I^\pi$  assignment comes from the  $p : 7/2[633] \otimes n : 3/2[622]$  configuration, where the  $2^+$  state is the triplet GM member. Our placement of this level is in agreement with the NDS compilation [6], as seen in fig. 2. The corresponding



**Fig. 2.** Expected low-energy bandheads in the  $Z = 155$  nucleus  $^{254}\text{Es}$  with  $E_x \leq 300$  keV. The bandhead energies are evaluated using the data listed in table 1. The evaluation methodology is discussed in the text.

singlet ( $K_S = 5^+$ ) state from this configuration, as yet unobserved, is evaluated to lie around 185 keV.

Moody *et al.* [4] have opined that  $\alpha$ -decays to bands arising from the  $p : 7/2[633] \otimes n : 7/2[613]$  coupling have lesser HF than those arising from the  $p : 7/2[633] \otimes n : 3/2[622]$  pairing. Experimental HF values for  $\alpha$ -feeding to the  $7^+$  g.s. band members are seen to be high [4,5]. It is therefore unlikely that the  $2^+$  isomeric state at 84 keV is fed directly in the  $\alpha$ -decay of  $^{258}\text{Md}$ .

The 448 keV  $8^-$  level in  $^{254}\text{Es}$  is identified as the triplet member of the  $p : 7/2^- [514] \otimes n : 9/2^+ [615]$  configuration. The very low HF (2.1) [5] of the  $\alpha$ -transition to this level from the parent  $^{258}\text{Md}$  is indicative of it being the analogous state in  $^{254}\text{Es}$ , having the same configuration as that of the g.s. in  $^{258}\text{Md}$ .

### 3.2 Unobserved low-lying levels

Apart from the  $0^+$  level from the  $(7^+, 0^+)$  GM pair and the  $5^+$  level from the  $(2^+, 5^+)$  GM pair, our evaluation identifies eight other as yet unidentified low-lying levels with  $I^\pi \leq 5$ . We briefly outline their structures in this subsection. The  $p : 7/2[633] \otimes n : 1/2[620]$  configuration results in the GM pair  $4^+$  and  $3^+$  at about 130 keV and 210 keV, respectively.

The GM pair of  $5^-$  at 145 keV and  $2^-$  at 194 keV result from the  $p : 3/2[521] \otimes n : 7/2[613]$  configuration. The  $p : 3/2[521] \otimes n : 3/2[622]$  configuration results in the triplet state  $0^-$  and the singlet state  $3^-$ . Presently, no experimental information is available on the Newby shift energy for this configuration. We hence estimate the value from a previous work on Newby shifts and  $E_{GM}$  for nuclei in the rare-earth regions and actinides [26]. Based upon the variation of the Newby shift term observed in the actinides and taking the correct parity notation into consideration we assume a Newby shift of 30 keV for this GM pair. We accordingly place the  $0^-$  level at 215 keV and the singlet  $3^-$  level above it at 295 keV. Finally, the  $p : 3/2[521] \otimes n : 1/2[620]$  combination gives a triplet  $2^-$  state at 224 keV and a singlet  $1^-$  level at 330 keV.

### 3.3 Characterisation of other levels observed in $^{258}\text{Md}$ $\alpha$ -decay

The NDS evaluators [5,6] question the  $6^-$  bandhead at 215 keV in  $^{254}\text{Es}$  arising from the  $p : 3/2[521] \otimes n : 9/2[615]$  combination, as proposed by Moody *et al.* [4] on the basis of two gammas related to this level—the 215 keV gamma to the ground state and the 189 keV gamma feeding this level from a higher level at 404 keV. The NDS evaluators argue that since the 215 transition to the g.s. with a configuration of  $p : 7/2[633] \otimes n : 7/2[613]$  involves a change in configuration for both the neutron and the proton orbitals, it would be highly hindered. Moreover, the  $3/2[521]$  proton single-particle state in  $^{253}\text{Es}$  lies at 105 keV while the  $9/2[615]$  neutron single-particle state in the  $N = 155$  isotone  $^{253}\text{Cf}$  is at 240 keV [27]. Going by our formulation outlined in the previous section, this would place the resulting singlet  $6^-$  level certainly above

350 keV. On these arguments the existence of the  $6^-$  level at 215 keV, with the assigned configuration, and its next rotational  $7^-$  member at 290 keV and  $8^-$  at 377 keV, cannot be supported.

Also, the level at 377 keV has been assigned by Moody *et al.* [4] as the  $8^-$  rotational member of the  $6^-$  band at 215 keV. However, the NDS compilation [5,6] suggests that an assignment of  $(7^+, 8, 9^+)$  to this level would be more consistent with the observed gamma decays to the  $(7^+, 8^+, 9^+)$  ground-state band members. They also pointed out the inconsistency of the  $8^-$  assignment with the observed low HF of 37 shown for the  $\alpha$ -feeding transition to the 377 keV level in  $^{254}\text{Es}$  from the parent  $^{258}\text{Md}$ . Available data [5,6] shows that the 297 keV gamma from this level to the  $8^+$  level at 80 keV is quite intense. The  $8^+$  member of the ground-state band appears from the  $p : 7/2[633] \otimes n : 7/2[613]$  configuration whereas the suggested  $8^-$  for the 377 keV level corresponds to the configuration  $p : 3/2[521] \otimes n : 9/2[615]$ . This would require the gamma transition between these levels to be highly hindered since a change in both the proton and neutron orbitals is called for. Our analysis shows that the  $8^+$  singlet band of the 2qp configuration  $p : 7/2[633] \otimes n : 9/2[615]$  may lie in the 350–400 keV range. If we assume the 377 keV level to have this structure then since the configuration of the decaying parent  $^{258}\text{Md}$  g.s. is  $p : 7/2[514] \otimes n : 9/2[615]$ , the  $\alpha$ -feeding from the parent to this level would now involve a change only in the proton orbital, hence making it a low hindrance  $\alpha$ -transition. Moreover, Moody *et al.* [4] have inferred that a transition between levels that involves a change in the proton configuration while retaining the neutron configuration would show a lower HF than a transition involving changing neutron orbitals and the same proton orbital. Based on these arguments, we propose that the level at 377 keV is the  $I^\pi K = 8^+ 8$  state with the 2qp configuration of  $p : 7/2[633] \otimes n : 9/2[615]$ .

The 404 keV level in  $^{254}\text{Es}$  is assigned  $(7^-, 8^-)$  by Moody *et al.* [4] based on the cascade of the 189 keV gamma from this level with the ground-state decaying gamma from the proposed level at 215 keV. The NDS evaluators neither place the the 189 keV gamma in their tabulation nor assign a possible structure to this level. This level also does not appear in their decay scheme. In our evaluation, we observe that the  $p : 7/2[514] \otimes n : 7/2[613]$  combination results in the singlet  $7^-$  level around 400–450 keV. However, Moody *et al.* [4] have advanced arguments that point to the conclusion that the hindrance factor for an  $\alpha$ -feeding from the parent  $^{258}\text{Md}$  g.s. would be the lowest for the analogous state in  $^{254}\text{Es}$ , followed by a transition to the  $7^-$  level from  $p : 7/2[514] \otimes n : 7/2[613]$  in  $^{254}\text{Es}$ . Currently, available data [4–6] shows that while the HF to the analogous state at 448 keV in  $^{254}\text{Es}$  is the least (2.1), the HF to the 404 keV level is the next lowest (HF = 10.4) [5]. We, hence, suggest that the level at 404 keV is the singlet  $7^-$  from the  $p : 7/2[514] \otimes n : 7/2[613]$  combination.

With their indicated intrinsic structures and in the light of the uncertainty associated with currently available experimental data, the observation of all the missing

low-lying states becomes a problem meriting a detailed investigation. One may possibly look for these low-lying, mostly low-spin structures, by trying to populate the  $^{254}\text{Es}$  levels through  $n$  capture in  $^{253}\text{Es}$  and/or transfer reactions like  $(t, p)$  on the  $^{252}\text{Es}$  target.

In summary, we have used a simple phenomenological model involving residual  $p$ - $n$  interactions to evaluate bandhead energies of low-lying states in the odd-odd nucleus  $^{254}\text{Es}$ . Apart from the  $7^+$  g.s. and the isomeric  $I^\pi = 2^+$  level at 84 keV, our calculations clearly indicate the possible existence of at least ten other previously unidentified levels with  $K^\pi \leq 5$  and energies  $E_x \leq 300$  keV. Our study has resulted in assigning the 2qp configurations and the excitation energies of all the ten missing levels. With input from the well-established levels in  $^{254}\text{Es}$  and available experimental data, the intrinsic configurations of other levels observed in the  $\alpha$ -decay of  $^{258}\text{Md}$  are re-examined. The  $6^-$  level at 215 keV suggested earlier is not supported in our formulation. We propose that the level at 377 keV be assigned  $I^\pi K = 8^+8$  with the 2qp configuration  $p : 7/2[633] \otimes n : 9/2[615]$ . The 404 keV level is suggested by us to be the singlet  $7^-$  state from the  $p : 7/2[514] \otimes n : 7/2[613]$  combination.

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